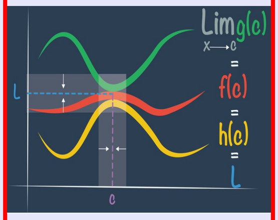
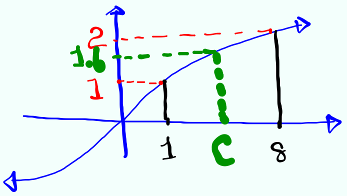
Calculus I Final Exam Review



Feb 19-8:47 AM

Find f_{ave} for $f(x)=\sqrt[3]{x}$ on $[1,8]$.



$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{ave}=\frac{1}{8-1}\int_1^8 \sqrt[3]{x}\,dx$$

$$=\frac{1}{7}\int_1^8 x^{1/3}\,dx$$

$$=\frac{1}{7}\left.\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right|_1^8$$

$$=\frac{1}{7}\left.\frac{x^{4/3}}{4/3}\right|_1^8=\left.\frac{3}{28}x\sqrt[3]{x}\right|_1^8$$

$$=\frac{3}{28}\left[8\sqrt[3]{8}-1\sqrt[3]{1}\right]$$

$$=\frac{3(16-1)}{28}=\frac{45}{28}\approx 1.6$$

$f(c)=1.6$

$\sqrt[3]{c}=1.6 \rightarrow c=1.6^3$

$c\approx\boxed{4.1}$

Dec 9-7:28 AM

Rotate the region bounded by $f(x) = -2x - x^2$ and x-axis by y-axis.

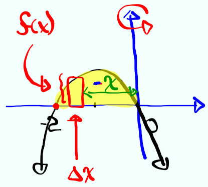
Parabola

$f(x) = 0$

$-2x - x^2 = 0$

$-x(2+x) = 0$

$x = 0,\ x = -2$



Shell Method

Ref. Rect. Parallel to A.O.R.

$$V = \int_{-2}^{0} 2\pi \cdot -x(-2x - x^2)\,dx$$

$$= 2\pi \int_{-2}^{0} (+2x^2 + x^3)\,dx$$

$$= 2\pi \left[\frac{+2x^3}{3} + \frac{x^4}{4} \right] \Bigg|_{-2}^{0}$$

$$= 2\pi \left[0 - \left(\frac{2(-2)^3}{3} + \frac{(-2)^4}{4} \right) \right]$$

$$= 2\pi \left[-\left(\frac{-16}{3} + \frac{16}{4} \right) \right]$$

$$= 2\pi \left[\frac{16}{3} - \frac{16}{4} \right] = 2\pi \cdot 16 \left[\frac{1}{3} - \frac{1}{4} \right]$$

$$= 32\pi \cdot \frac{1}{12} = \boxed{\frac{8\pi}{3}}$$

Dec 9-7:36 AM

Rotate by y-axis the region bounded by

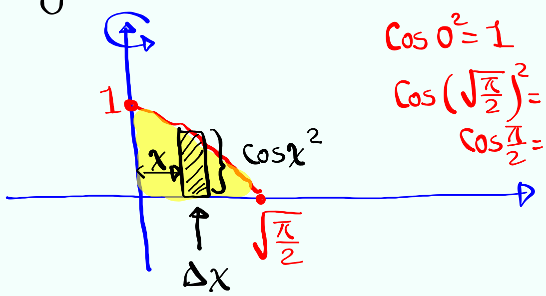
$y = 0$, $x = 0$, $y = \cos x^2$ for $0 \le x \le \sqrt{\frac{\pi}{2}}$.

x-axis, y-axis

$\cos 0^2 = 1$

$\cos\left(\sqrt{\frac{\pi}{2}}\right)^2 = \cos\frac{\pi}{2} = 0$

Shell Method



$$V = \int_{0}^{\sqrt{\frac{\pi}{2}}} 2\pi x \cos x^2\,dx$$

$u = x^2$

$du = 2x\,dx$

$x = 0 \to u = 0$

$x = \sqrt{\frac{\pi}{2}} \to u = \left(\sqrt{\frac{\pi}{2}}\right)^2 = \frac{\pi}{2}$

$$V = \int_{0}^{\pi/2} \pi \cos u\,du$$

$$= \pi \sin u \Big|_{0}^{\pi/2} = \pi\left[\cancelto{1}{\sin\frac{\pi}{2}} - \cancelto{0}{\sin 0}\right] = \boxed{\pi}$$

Dec 9-7:52 AM

$$\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$$

Def. of derivative

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$a = \frac{\pi}{3}$

$f(x) = \cos x \longrightarrow f'(x) = -\sin x$

$f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$

Evaluate $\displaystyle \lim_{x \to \frac{\pi}{4}} \frac{\overbrace{\tan x - 1}^{f(x) - f(a)}}{\underbrace{x - \frac{\pi}{4}}_{x - a}} = f'\left(\frac{\pi}{4}\right)$

$= \sec^2\left(\frac{\pi}{4}\right)$

$= \left[\sec\frac{\pi}{4}\right]^2$

$= \left(\sqrt{2}\right)^2 = \boxed{2}$

$a = \frac{\pi}{4}$

$f(x) = \tan x$

$f(a) = f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

Dec 9-8:00 AM

$\sqrt{x} + \sqrt{y} = 3$ Find $\left.\frac{dy}{dx}\right|_{(1,4)}$

$x^{1/2} + y^{1/2} = 3$

$\frac{d}{dx}\left[x^{1/2} + y^{1/2}\right] = \frac{d}{dx}[3]$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$

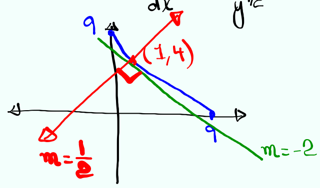
multiply by 2 $\quad x^{-1/2} + y^{-1/2}\frac{dy}{dx} = 0$

$y^{-1/2}\frac{dy}{dx} = -x^{-1/2}$

$\frac{dy}{dx} = -\frac{x^{-1/2}}{y^{-1/2}} \longrightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

$\left.\frac{dy}{dx}\right|_{(1,4)} = -\frac{\sqrt{4}}{\sqrt{1}}$

$= \boxed{-2}$



$\sqrt{x} + \sqrt{y} = 3$

$y - 4 = -2(x - 1)$

$\boxed{y = -2x + 6}$

Tangent line at (1,4)

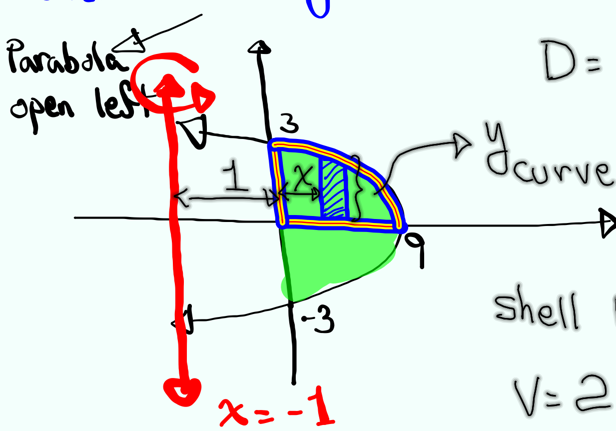
$y - 4 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x - \frac{1}{2} + 4$

$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$ Normal line at (1,4)

Dec 9-8:07 AM

Rotate the region bounded by $x=0$, (y-axis) and $x=9-y^2$ about $x=-1$.



$D = x+1$

$y^2 = 9-x$

$y = \sqrt{9-x}$

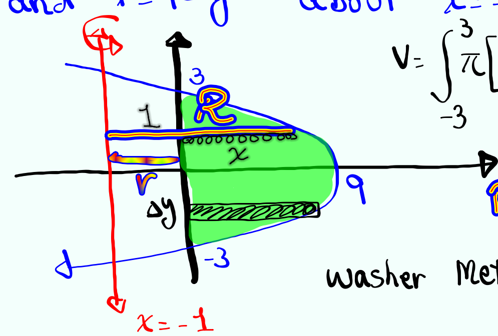
Shell Method

$$V = 2\int_0^9 2\pi(x+1)\sqrt{9-x}\,dx$$

Dec 9-8:17 AM

Rotate the region bounded by $x=0$, (y-axis) and $x=9-y^2$ about $x=-1$.

$$V = \int_{-3}^{3} \pi\left[R^2 - r^2\right] dy$$



$R = x_{curve} + 1$

Washer Method $= 9-y^2+1$

$= 10-y^2$

$r = 1$

$$V = \int_{-3}^{3} \pi\left[(10-y^2)^2 - 1^2\right] dy$$

$$= 2\int_0^3 \pi\left[100-20y^2+y^4-1\right] dy = 2\pi\left[99y - \frac{20y^3}{3} + \frac{y^5}{5}\right]_0^3$$

Dec 9-8:17 AM

Evaluate $\int_0^1 3x^2\,dx$ using $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\,\Delta x$.

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$ $\quad x_i = 0 + i \cdot \Delta x = \frac{i}{n}$

$f(x_i) = 3\left(\frac{i}{n}\right)^2 = \frac{3i^2}{n^2}$ $\quad \sum_{i=1}^{n} \frac{3i^2}{n^2} \cdot \frac{1}{n}$

$= \frac{3}{n^3} \sum_{i=1}^{n} i^2 = \frac{3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$

$$\lim_{n\to\infty} \frac{3n(n+1)(2n+1)}{6n^3} = \lim_{n\to\infty} \frac{6n^3 + \text{Junk.}}{6n^3}$$

$= \frac{6}{6} = 1$ ✓

$$\int_0^1 3x^2\,dx = \left.\frac{3x^3}{3}\right|_0^1 = \left.x^3\right|_0^1 = 1^3 - 0^3 = \boxed{1}$$ ✓

Dec 9-8:31 AM